Summary of Proposed Research Program for Doctor of Philosophy

Title

The Use of Graphics Calculators and other Teaching Approaches to Enhance the Learning of Vectors in Year 11 Mathematics.

Abstract

The proposed research will inquire into students' learning of vectors in the presence of students' personal ownership of graphic calculators. Use of technology in my own teaching combined with research experience through my Master's project (Forster, 1997) provide background to the research which will take place in the context of fast-growing appropriation of technology for mathematics education. The research will involve prolonged observation of a Year 11 Geometry and Trigonometry class and concerns the cognition of vector concepts with learning seen as both a personal and social activity. The influence of graphic calculators on students' learning outcomes will be a focus of the inquiry. A hermeneutic phenomenological research approach will be adopted with the orientation towards the learning experiences of individual students. Frameworks for analysis of classroom activities will include constructivism and van Hiele's (1986) model for cognitive development, and for students' learning outcomes will include the SOLO taxonomy (Biggs & Collis, 1989). Guba and Lincoln's (1989) quality criteria appropriate to a qualitative research paradigm will guide the research.

Objectives

The purpose of the proposed research is to identify teaching approaches, which may utilise graphics calculators, and which seem to promote students' deep understanding of vector relationships. The metaphors of "curriculum as content ... curriculum as a program of planned activities ... curriculum as intended learning outcomes ... curriculum as experience" (Schubert, 1986, p. 26-31) provide a framework for the study.

Content
The content is stated in the syllabus for the Year 11 Geometry and Trigonometry course (Curriculum Council, 1998). The history of the introduction of the study of vectors into the West Australian mathematics curriculum will be explored. My interest is in the relevance of vectors to students and the theoretical basis for expecting that Year 11 students are ready to learn the sometimes abstract and complex vector concepts.

Prior knowledge for satisfactory entry into studying vectors as part of the Geometry and Trigonometry course, and the prior school experience that incoming Year 11 students can reasonably be expected to have relevant to the study of vectors, will be established. The purpose is to ascertain any possible disjunctures between the necessary and actual prior knowledge of students.

Classroom activities
A class will be observed for a prolonged period of time while learning about vectors. Students' involvement in lessons, how students make sense of instruction, students' connection of the (concrete) graphical representation of vector relationships on graphics calculators to algorithms, students' recognition of vector relationships in real-life and ability

to formulate the relationships with mathematical terminology will be subject to inquiry. In addition, the teaching method will be of prime interest including the assumptions made about students' prior knowledge, the sequencing and teaching approach to content, the social organisation of the classroom and the use of graphics calculators. The purpose in making classroom observations is to describe students' learning experiences from a cognitive perspective.

Learning outcomes
In order to make and give plausibility to my claims for what might comprise effective teaching, students' learning outcomes will be evaluated. This will involve both informal evaluation, eg. through questioning students as they work, and formal evaluation, e.g. statistical analysis of test results. Students' misconceptions and difficulties in understanding (including those consequential to the use of graphics calculators) will be identified and possible sources of these problems explained.

Experience
Students' experience in the classroom will be central to the inquiry, i.e. the orientation of the inquiry is towards the students. The research concerns cognition of vector concepts with learning seen as both an individual and social activity (Ernest, 1995). Interpretation of students' experience will rely on the classroom observations, student interviews and analysis of learning outcomes, with the focus being on four individual students.

The research questions are:
1. What is involved in students' development of deep understanding of vector concepts?
2. How might use of graphics calculators enhance students' learning of vector concepts?

Background

The National Statement for Mathematics (Australian Education Council, 1991) recommends: "take full advantage of the potential of calculators for mathematics within the total curriculum" (p. 23). A major learning outcome specified in The Curriculum Framework for Western Australia is: "Students select, use and adapt technologies" (Curriculum Council, 1997, p. 17). Graphics calculators will, for the first time, be used by candidates in West Australian tertiary entrance examinations in 1998. In preparation for this innovation there has been widespread adoption of graphics calculators in West Australian secondary schools in 1997. These initiatives indicate that while slow compared to the uptake of technology in the business community the impact of technology on mathematics education is not insignificant.

My Year 11 and 12 students have used computer graphing packages since 1987 as a regular part of classroom activities, and my use of computers for teaching advanced in 1992 when the dynamic geometry software Cabri Géomètre became available. My Master's action-research study (Forster, 1997) involved the design and implementation of four technology-based mathematics investigations, and evaluation of students' learning outcomes. The study enhanced my understanding of both the potentials and the pitfalls of teaching and learning with technology. However, even in light of these experiences, the impact on the curriculum of my students having their own graphics calculators in 1997 was far greater than I had anticipated — using technology routinely is different to utilising it sporadically.

The situation in Western Australia parallels the world-wide adoption of technology in mathematics education. Research programs are inquiring into and informing the reform.

Subjects of inquiry have included new approaches to traditional areas of content such as algebra (ASP, Dowsey, Stacey & Tynan, 1995), calculus (Confrey & Doerr, 1996) and geometry (Balacheff, 1993); new ways of thinking, e.g., computer aided visualisation (Kawski, 1997); theory-based educational mathematics software (Steffe, 1998), students' belief in the infallibility of computer generated results (Lapp, 1998), gender issues associated with technology (Apple, 1988), misconceptions propagated by technology (Goldenberg, 1988), students' resistance to exploratory method (Kent, Ramsden & Wood, 1996), staff development needs (Goodell & Frid, 1997) and teacher epistemology in computerised classrooms (McDougall, 1996). These references indicate the scope of current research into using technology for teaching/learning mathematics. The research, however, seems to predominantly involve higher-level technologies than the graphics calculators recommended by the Curriculum Council and used in West Australian schools.

The study of vectors was introduced into the West Australian curriculum in 1992, and is a major component of the Year 11 Geometry and Trigonometry course (Curriculum Council, 1998). In my experience of teaching the course (without technology), students had difficulty understanding many of the vector concepts.

In summary, use of technology in my own teaching and research experience through my Master's project provide background to the proposed research. The research takes place in the context of fast-growing adoption of varied levels of technology for mathematics education. It will inquire into the learning of vector concepts, an endeavour with which my students in the past had difficulty.

Significance

The proposed research gives me an opportunity to identify and explain students' successes and difficulties in understanding vectors which might lead to the development of sound teaching practices. For example, identification of inadequate prior knowledge will inform the Year 7-10 mathematics programming at the college which is the site of the research. In a wider domain, the research may inform syllabus revisions for upper secondary courses in Western Australia.

The parametric equation facility on graphics calculators affords students the opportunity to link graphical and symbolic forms of vectors, together with a table of co-ordinate values. The calculators can also be used with distance data-logging probes for "real-life" vector applications. The research allows me to explore and evaluate visual/symbolic/numeric methods and "real-life" applications using graphics calculators for teaching/learning vectors, which will contribute to the growing understanding of how to effectively appropriate technology for mathematics education.

The greatest significance of a thesis can be to the researcher him/herself as I experienced for my Master's project (Forster, 1997). Aside from meeting the requirement of doctoral research to make a substantial and original contribution to the subject area being investigated, I anticipate that my research will be a personally rewarding learning experience. Other potential benefits of the research will be professional development for the teacher-participant and heightened self-awareness of their learning by the student-participants.

Research Method

Sample selection
Observations will take place in an urban, private girls' college in Western Australia. The study will involve one Year 11 class of between 20 and 25 students. The teacher is male, has long teaching experience, and is recognised to have good rapport with his students. Four students of varying academic ability will be selected for interview and close observation.

Data Generation
Data will be generated predominantly through naturalistic inquiry which aims "[t]o study a situation without manipulation, and with minimum imposition or constraints. The data are usually derived from observation and interviews, but could come from artefacts, memorabilia, or other non-intrusive measures" (Welch, 1983, p. 96). Intervention will involve encouraging, but not imposing, the use of technology in the teaching method. The teacher will be supported with provision of graphics-calculator-based curriculum materials, if he so chooses.

The history of the introduction of vectors into the curriculum will be established through interviewing those who were involved in the decision and by perusing relevant documents. Prior knowledge required to study the vector topic will be determined by considering the Geometry and Trigonometry syllabus and will be informed by my experience of teaching the course, and will be subject to validation by the teacher-participant. Year 7-10 mathematics curriculum documents will be examined to establish students' likely prior knowledge. "Curriculum documents" are taken to mean the teaching programs in the private school which will be the site of the research, and the student-learning-outcome statements of the Education Department of Western Australia.

Qualitative data from the in-class phase of the research will comprise classroom observations, recorded in field notes and videotaped; informal discussions with the teacher-participant and students recorded afterwards in a journal; audiotaped interviews with teacher and student participants. In addition, work samples, samples of assessment tests and other curriculum materials, with students' answers, will be collected. Quantitative data will include assessment test scores and responses to the Constructivist Learning Environment Survey CLES (Taylor, Fraser & White, 1994). The CLES will be carried out at the conclusion of classroom observations for the study.

The syllabus for the Geometry and Trigonometry course recommends forty hours class-time be spent on the study of two-dimensional vectors and twenty hours for three-dimensional vectors. I will observe and video-tape lessons on vectors, choosing lessons (a) covering the basic principles of vectors or (b) involving special use of technology including graphics calculators or (c) involving applied problem solving including practical applications. The remainder of the lessons will be videotaped.

Informal discussion with the teacher will take place after each observed lesson and the teacher will be interviewed once in each week that classroom observations are made (vectors are about half the Geometry and Trigonometry course). Students will be questioned during lessons, to impinge minimally on the students' lesson involvement, and the focus students will be interviewed several times during the year. Interviews will be used to clarify my own emerging understandings, to seek alternative views from the participants, and to generate additional data from participants' responses to my written interpretation of events.

Data Analysis

An interpretative research approach will be used. Candy (1989) describes the assumptions of interpretative theory:

> (1) The belief that any event or action is explicable in terms of multiple interrelating factors.
> (2) An acceptance of the extreme difficulty in attaining complete objectivity.
> (3) The view that the aim of inquiry is to gain an understanding of individual cases, rather than universal laws or generalisations.
> (4) The assumption that the world is made up of tangible and intangible multifaceted realities.
> (5) A recognition that inquiry is always value laden. (p. 4)

Data will be analysed on a continuous basis from the start of classroom observations and, in accordance with a constructivist research paradigm (Noddings, 1990), the understandings will feed iteratively into the ongoing data generation and analysis.

Hermeneutic phenomenological interpretation (Van Maanen, 1990) of lived experience will provide a framework for the analysis. The phenomenon subject to inquiry is "students' development of deep understanding of vector relationships". The empirical data will be the text for hermeneutic (interpretative) analysis, where text is seen in thought and (sometimes videoed) action as well as in the data records of observations (field notes, interview transcripts etc.). The text of the prolonged data gathering will be analysed to identify significant factors, isolated or repeatedly evident, to students' effective learning.

Referents for the analyses of classroom activities will be constructivism, van Hiele's model (1986) of intellectual development and the literature on technology-based learning and visual learning. Referents for evaluating students' learning outcomes will be the SOLO taxonomy (Biggs & Collis, 1989) and the learning objectives stated in the Geometry and Trigonometry syllabus (Curriculum Council, 1998). The design and evaluation (after implementation) of any innovative graphics-calculator-based curriculum materials will be informed by discussion with Curtin University staff who have investigated visual, computer-based learning of vector concepts with their first-year students.

Students' assessment test scores will be summarised statistically but the measures will be used to contribute to understanding individual students' experience rather than being considered important measures of class performance. Similarly the CLES (Taylor, Fraser & White, 1994) responses will be analysed to contribute to understanding individual student viewpoints rather than been seen as a whole class measure. These approaches reflect the nature of the qualitative inquiry being undertaken — the interest is in the cognition of individuals, and not in the prevalence of characteristics across a population.

Theoretical frameworks

Constructivism is a theory about learning with the basic premise that "all knowledge is constructed". An implication of constructivism is that knowledge is a personal construct subject to scrutiny, so that "truth can never be claimed for knowledge" (Von Glaserfeld, 1990, p. 19). There are different forms of constructivism but the two used for this research project will be radical constructivism (Von Glaserfeld, 1990) and social constructivism (Ernest, 1995). Von Glaserfeld's (1990) principles of radical constructivism are:

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognising subject.
2. (a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability.
   (b) Cognition serves the subjects organisation of the experiential world, not the discovery of an objective ontological reality. (p. 19)

Social constructivism is inclusive of radical constructivist principles including that cognition is personal, but at the same time learning is viewed as a social activity (Ernest, 1995). The world is seen to be "socially constructed" (Ernest, 1995, p. 480).

Constructivism will be used as a referent for analysing planned and actual classroom activities, and for the research method. Using constructivism as a referent for teaching involves asking "are students being provided with opportunities to construct their own knowledge?". A constructivist approach also involves making explicit to students that their knowledge, like everybody's, is a personal construct subject to revision and that knowledge does not exist as "fact out there for the taking" — that they need, and have a responsibility, to be actively involved in their own learning. With constructivism as a referent for research method, each stage of the research is seen to iteratively inform the next stage. The method is an emergent construction, with research practices not set in advance, and is pragmatic, strategic and self-reflexive (Denzin & Lincoln, 1994).

Van Hiele's (1986) model for cognitive development recommends that new concepts be developed through:

- initial exploratory activities to establish the background or prior knowledge of the student, using the students' language. The existing schema might be resistant to change or development.
- simple exercises to establish the direction of instruction and the breadth of the field, together with establishing appropriate language.
- further social development of the topic using the new language with peers and the teacher. This moves individual personalised expressions concerning content, to one that is mutually shared within the classroom.
- solving non-routine problems in practical contexts with multiple solution paths (to develop more comprehensive mental schema).
- summary of information for consolidation and to set the scene for the next level of knowing.

The SOLO taxonomy (Biggs & Collis, 1989) for cognitive development suggests that learning occurs in one of five modes: sensori-motor, iconic, concrete symbolic, formal (engaging in abstract thought), post formal (able to challenge theories).
Within each mode there are several levels

- unistructural: the student only works with one piece of relevant information
- multistructural: two or more pieces of information are used, but the connection between them is not perceived by the student. No integration occurs. Inconsistencies may be apparent.
- relational: all data is used and made into a coherent structure with no inconsistencies. eg. relational understanding in the concrete symbolic mode would mean a student could

understand two-dimensional vector motion when viewing the "vector components and resultant" dynamically displayed on a graphics calculator.

Quality criteria
Guba and Lincoln (1989) suggest criteria for legitimating qualitative research and these will guide the research. Legitimacy concerns how well research represents (to the reader) the reality of events as respondents saw them. To satisfy the criterion of *credibility*, prolonged engagement for the main part of the study, with persistent observation (to avoid delusion), and discussion with the teacher to find his interpretation of events (to avoid bias on my part) will be undertaken. My aim is not to pursue mutual agreement between the teacher and myself but to discuss and explain our perceptions in terms of our (differing) beliefs and values, i.e. to achieve *educative authenticity* through both learning off, but not necessarily agreeing with, each other. Following Taylor's (1997) suggestion that educative authenticity might not be appropriate if discussion would be counterproductive, e.g. if it might alienate or demoralise a teacher, educative authenticity will be suspended if warranted by an ethic of care (Noddings, 1984). The research report will give voice to all participants including when their viewpoints differ from mine.

In adherence to Guba and Lincoln's (1989) recommendations, thick description of the context and culture of the inquiry will be provided to enable the reader to judge the *transferability* of the results to his/her own situation. *Dependability* for data will be achieved through reporting the research process in a way that the data generation can be tracked by a reader, while *confirmability* of the reported data will be assured through recording its sources and interpretation process. The reporting will also be guided by Denzin and Lincoln's (1994) criteria "verisimilitude (is it believable), personal responsibility, an ethic of caring, — multivoiced texts, and dialogues with subjects" (p. 5).

Multiple sources of data (teachers, students and myself) and different forms of data including interpretative data (observations, discussions, interviews, curriculum materials and students responses to them, field notes, journal) and numerical data (CLES responses, student test scores) will contribute to the quality of the research.

Ethical Issues

The teacher will not go into the study uninformed about his involvement in it--we have already discussed the proposed research activities and there will be further discussions near the commencement of classroom observations. The teacher will also be informed in writing about the nature of the project and his written consent to participate will be obtained. Informed, written consent will be secured from the principal of the college which is the site of the research.

The purpose of the research and students' involvement in it will be explained to the students in class, and in a letter to their parents. The written consent of the parents will be obtained.

I value the privilege of conducting research in a teacher's classroom and it is important to me that the teacher does not feel that I abuse that privilege. Dawson and Taylor (1998) commend that "A rich communicative relationship harbours an intellectual space for critique and thus offers potential for mutual growth, rather than the slumber of admiration or the anaesthesia of misplaced silence" (p. 12). Mutual growth and understanding was an outcome when a colleague and I critiqued each other's teaching as part of my Master's action-research project

(Forster, 1997). I hope for the same outcome in the proposed research study and see it as contingent upon a relationship of mutual trust.

Ethics relevant to a constructivist perspective are described by Guba and Lincoln (1989) and will guide the study. These *authenticity criteria* include *fairness*, which involves identification, opinion seeking, and negotiation of understanding (including conflict) with all stakeholders. In the event that educative authenticity (Guba & Lincoln, 1989) is suspended, i.e. if I don't discuss my (written) interpretation of an event with the teacher, I will ensure to have sound justification i.e. for not doing so and be alert to bias in my analysis--any critique will be grounded in the theoretical frameworks. *Ontological authenticity* "refers to the extent to which individual respondents' own emic constructions are improved, matured, expanded and elaborated, in that they now possess more information and have become more sophisticated in its use" (Guba & Lincoln, 1989, p. 248). This requires individual stakeholders to become more informed through the research. *Catalytic authenticity* (Guba & Lincoln, 1989, p. 249) requires stimulation of further action through the research.

The teacher, students and the site of the research will not be identified in the dissertation, and it will be made clear to the participants that they have the right to not answer questions and that they may withdraw, without prejudice, from the research at anytime.

Facilities and Resources

Audio-recorder, video-recorder, computer software (qualitative-data-base) will be required to complete the study.

Data Storage

The data storage provisions are outlined in the attached Research Data Management Plan and meet the Curtin University Research Data and Primary Materials Policy.

Time Line

Year 1	Commence the literature review for the research.
Year 2	Continue the literature review. Generate and interpret data as a continuous process throughout the year. Commence drafting the report.
Year 3	Complete the requirements for the degree before the end of the year.

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